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Analysis of Orthotropic Plates with Continuous Boundary edge Using Finite Difference Method

Ziad H. Abo Mustafa
Almergb university - Libya
ziadslman2019@gmail.com

الملخص

البلاطات الخرسانية والصفائح القشرية عبارة عن عناصر إنشائية التي قد تكون على هيئة بلاطات أسقف أو أساسات حصيرية في المباني وكذلك قد تكون أجزاء من سفن أو طائرات أو أجزاء من آلة. البلاطات تتكون من مواد موحدة الخواص وأخرى غير موحدة الخواص وأحيانا تكون من مواد صفائحية مركبة. كذلك تصنف البلاطات من حيث السمك بالبلاطات الرفيعة ذات الهبوط البسيط والبلاطات الرفيعة ذات الهبوط الكبير وأيضا البلاطات السميكة.

تتطرق هذه الورقة إلى تحليل البلاطات غير الموحدة الخواص عند الحدود المستمرة وذلك باستخدام طريقة الاختلاف المتناهي finite difference method كوسيلة للحل. المعادلة التفاضلية العامة للبلاطات غير الموحدة الخواص من الدرجة الرابعة التي تم استنتاجها و معاملاتها باستخدام الشبكة غير متساوية الفراغات التي أنشأت للبلاطة، كما تم إعداد برنامج كمبيوتر لحل المعادلة التفاضلية المستنتجة للبلاطة المعرضة لأحمال مختلفة باختلاف نسبة بواسن ومعامل المرونة في الاتجاهين السيني والصادي. قد كان لتغيير نسبة بواسن تأثير مهم في عزم الانحناء على هذه البلاطات. في نهاية هذا البحث تم توضيح جداول ومنحنيات يمكن استغلالها لإيجاد قيم قوى العزم والإجهادات بهذه البلاطات إذا ما أراد المهندس الاستفادة منها.

ABSTRACT:

Plates are structural members that can be a slab, raft foundation in building. They can be also part of vessels, airplanes or parts of some machines. Plates can be formed by isotropic materials or

orthotropic materials or sometimes laminated composite materials. Plates are flat structural elements for which the thickness is much smaller than the other dimensions. Plates may be classified into three groups (1- Thin plate with small deflection 2- Thin plate with large deflection 3- Thick plate).

This paper is devoted to the analysis of orthotropic rectangular plates of continuous boundaries using finite difference method as a tool for such analysis. General forth order differential equation for orthotropic plate equation and its corresponding coefficient patterns of unequal spacing mesh were developed for the plate. A computer program was prepared and used to analyze the plates subjected to different loading with the variation of different Poisson's ratio and different modulus of elasticity in x&y directions. It was found that the value of deflection decreases when the Poisson's ratio increases. It was concluded that the Poisson's ratio has a significant effect on bending moment of the plate. More sets of conclusions will be listed by illustrating them in tabulated format and / or graphical diagrams that can be helpful to the engineers that need to benefit from these contribution.

KEYWORD: Raft Foundation, Finite Difference, Rectangular Plates, Orthotropic Plate, Boundary Condition.

INTRODUCTION:

Since Navier[1] in 1820 solved Lagrange equation of the isotropic plate bending problems employed the double Fourier series for a load and deflection relationships. Later an important approach was developed by M. Levy[1] in 1900. When his solution is compared with the Navier's method instead of a double series he dealt with a single series. Other important solution for the problems of plate bending is the energy method by Ritz[1].

Many references were assigned as text books for plate analysis such as Timoshenko[2], but with the high speed of personal computers, orthotropic plates began to grasp engineering attention

looking for exact analysis of plates to reach better economy of material usage and high accuracy.
In literature review solution of an orthotropic simply supported rectangular plate with uniform distributed load was illustrated in details[2,3].

This paper is devoted to the analysis of continuous boundaries for rectangular orthotropic plates using finite difference method as analysis tool. General forth order differential equation for orthotropic plate equation and its corresponding coefficient patterns of unequal spacing mesh were developed. A computer program was prepared and used to solve plates with different loadings with variations in Poisson's ratio and different modulus of elasticity in x&y directions[4].

FINITE DIFFERENCE METHOD:

The method of finite differences is a tool to write sets of equations for a specific purpose. In our case solution of Lagrange equation is modified for an orthotropic plate as shown in Eq. (1).

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \qquad (1)$$

$$H = D_{xy} + 2G_{xy} \qquad (2)$$

Where

D_x, D_y are flexural rigidities of an orthotropic plate.
G_{xy} torsional rigidity of an orthotropic plate.
q, intensity of load per unit area.

This method is applied here to replace the plate governing differential equation and the expressions defining the boundary conditions with equivalent difference equations. The solution of the bending problem thus reduces to the simultaneous solution of a set of algebraic equations written for every nodal point within the plate.

Coefficient Patterns of Rectangular Mesh for Orthotropic Plate:

The coefficient patterns of rectangular mesh are diagrammatically shown in Figure 1.

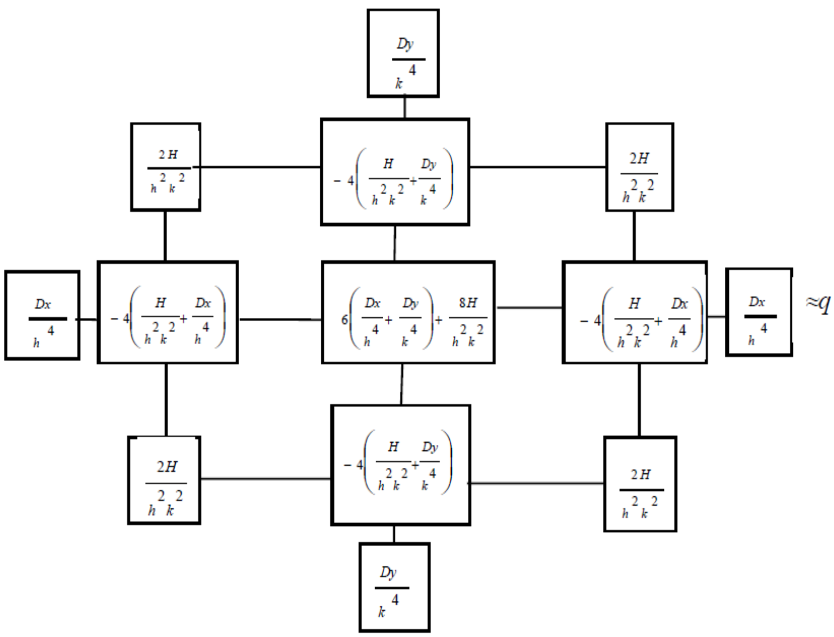

Figure 1. Coefficient patterns of rectangular mesh for orthotropic plate[1]

Rectangular Plate Pattern for Continuous Boundary Edges:

The following Figure 2. Shown a rccangular plate for continuous boundary edges. Two edges are simply support and the other two edges are fixed support.

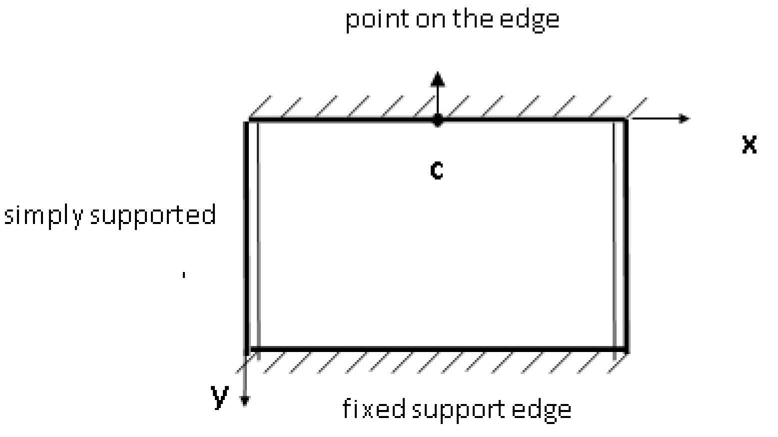

Figure 2. Rectangular Plate Pattern for Continuous Boundary Edges

Typical Problem for Orthotropic Plate:

Consider a simply supported orthotropic rectangular plate for all edges as shown in Figure 3. of side dimensions of a × b where a = 6 m, b = 4 m, subjected to a uniform load of q per unit area equal 10 KPa, plate thickness equal 0.12 m, , modulus of elasticities in x&y direction are Ex = 18 GPa and Ey = 12 GPa, and the Poisson's ratio in x&y direction are ν_x = 0.3and ν_y = 0.2 are considered. A computer program named Ortho. was developed for computing the deflections w, bending moments M_x , M_y, and stresses σ_x , σ_y at a grid of points with an even number of intervals along each side of the rectangular plate. tables 1 show the central deflection, central bending moments in x&y directions, central stresses in x&y directions. Results and accuracy convergence were demonstrated and are given in the form of tables and graphs. Discussion of these results are listed in below. Four different number of elements will be used for the problem.

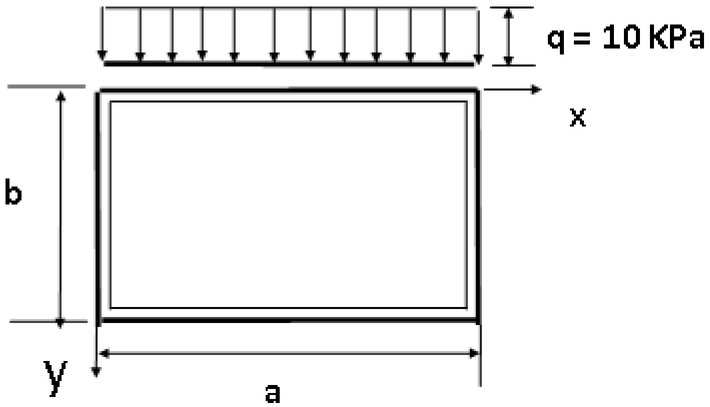

Figure 3. Orthotropic rectangular plate simply supported at all sides subjected to a uniform distributed load.

Convergence test using finite difference method:

The following table 1. Shows the results of orthotropic plate simply supported at all sides under uniform distributed load. W, M_x , M_y, σ_X , σ_Y at center of the plate.

Orthotropic factors { Ex, Ey, υ_x , υ_y }.

Table 1. Results of problem at center of plate.

Number of Grid spacing Nx × Ny	Total number of elements	W at center m	M_x at center N.m	M_y at center N.m
4 × 4	16	0.88357E-02	0.83350E+04	0.10182E+05
8 × 8	64	0.88900E-02	0.85479E+04	0.10567E+05
12 × 12	144	0.88986E-02	0.85896E+04	0.10642E+05
18 × 18	324	0.89041E-02	0.86104E+04	0.10678E+05
Analytical solution		0.89106E-02	0.87328E+04	0.10767E+05

Number of grid spacing Nx × Ny	Total number of elements	σ_X at center Pa	σ_Y at center Pa
4 × 4	16	0.34729E+07	0.42426E+07
8 × 8	64	0.35616E+07	0.44028E+07
12 × 12	144	0.35790E+07	0.44340E+07
18 × 18	324	0.35877E+07	0.44490E+07
Analytical solution		0.36387E+07	0.44863E+07

The following Figure 4. Shows the percentage of error for variation the number of element for deflections and bending moments in x&y directions at a center of orthotropic simply supported plate at all sides.

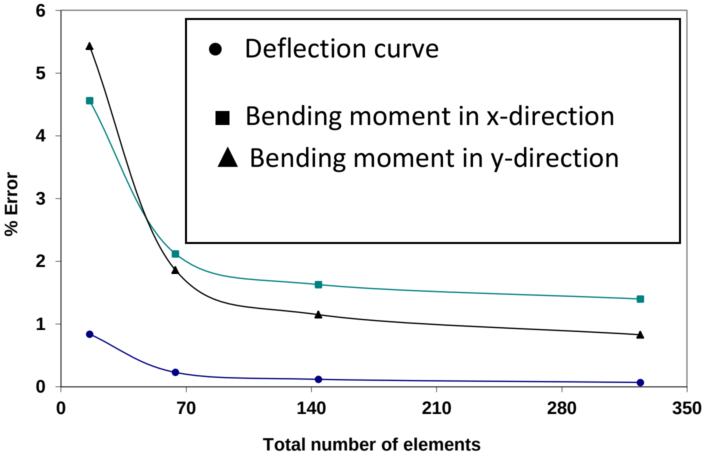

Figure 4. Convergence of deflections and bending moments in x&y directions at center of orthotropic simply supported plate at all sides subject to uniform load with variation of number of elements. (Finite difference Vs. Analytical solution).

Figure 4 Indicates that for 16 elements the percentage of error of central deflection is equal 0.84 % and is reduced to 0.23 % for 64 elements and to 0.12 % for 144 elements and 0.07 % for 324 elements. It is clear that as the number of elements are increased the percentage of error is decreased significantly and convergence is smooth, also in this figure indicates for 16 elements the percentage of error of central bending moment in x-direction is closer to 4.6 % this error is reduced until reach 1.4 % for 324 elements, while the curve of central bending moment in y-direction indicates that for 16 elements the percentage of error 5.4 % which is more than the error for central bending moment in x-direction for 16 elements, but at 324 elements the percentage of error is

0.83 % which is lesser than the percentage of error for the central bending moments in x-direction for 324 elements.

Typical seven problems results for Number of grid spacing Nx × Ny equal 324:

In this section seven problems of orthotropic rectangular plates as shown in Figure 5. with the same dimensions a = 6 m, b = 4 m, t = 0.12 m, and the same properties Ex = 18 GPa, Ey = 12 GPa, ν_x = 0.3, ν_y = 0.2. But with different boundary conditions and different types of loadings will be illustrated below by using computer program named ortho. that was made particularly for this study, number of grid spacing Nx × Ny equal 324 elements will be used for each problem. table 2. will show the central deflection, central bending moments in x&y directions, central stresses in x&y directions, and the bending moments at the center of the fixed edge of the plate. Results and accuracy convergence were demonstrated and are given in the form of tables and graphs. Discussion of these results are listed in this paper.

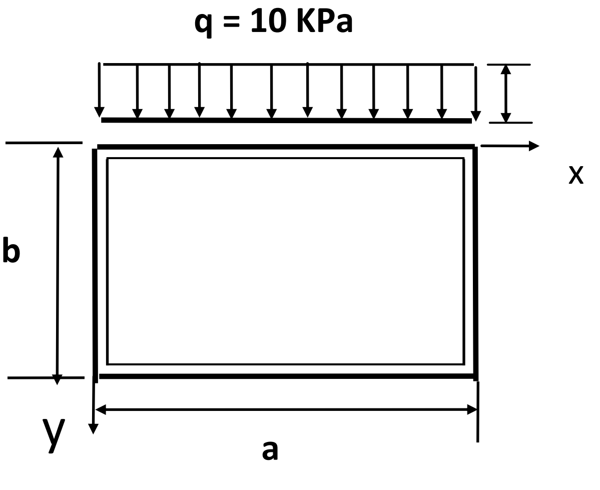

Figure 5. Case1. simply supported orthotropic rectangular plate subjected to uniform load of q per unit area..

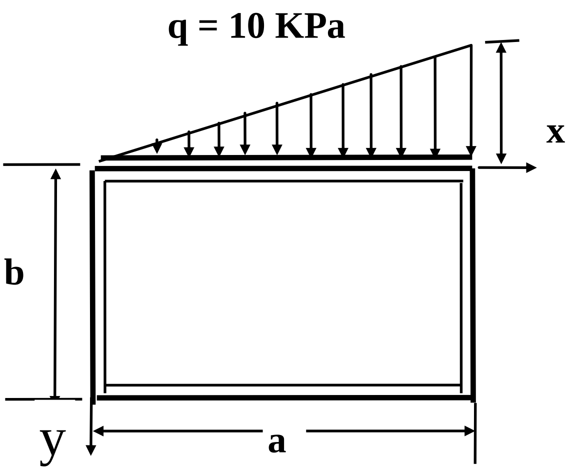

Figure 5. Case 2. simply supported orthotropic rectangular plate subjected to triangular load of q per unit area.

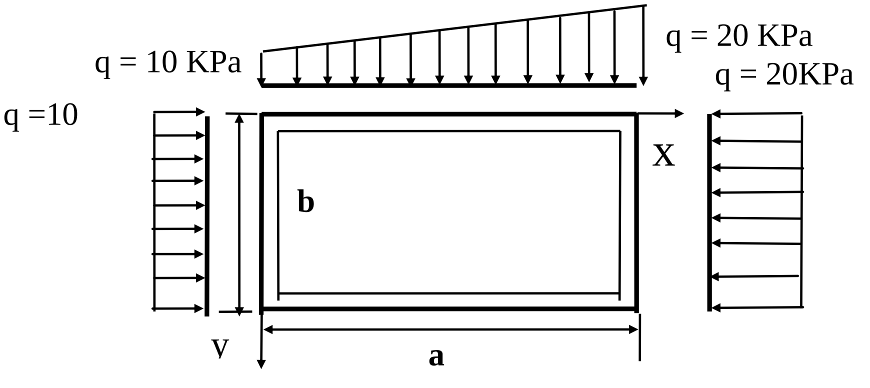

Figure 5. Case3. simply supported orthotropic rectangular plate subjected to linear trapezoidal load of q per unit area.

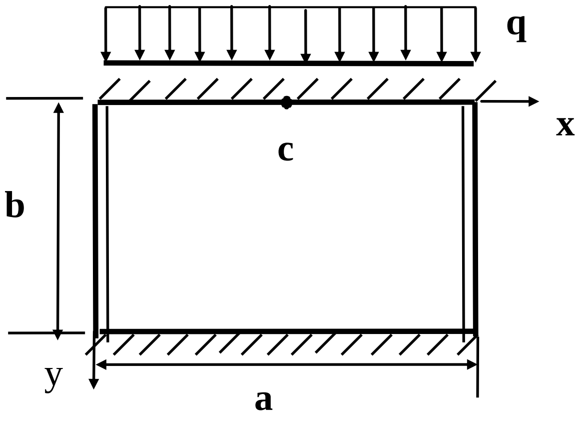

Figure 5. Case 4.Two edges simply support and two edges fixed subjected to uniform load of q per unit area.

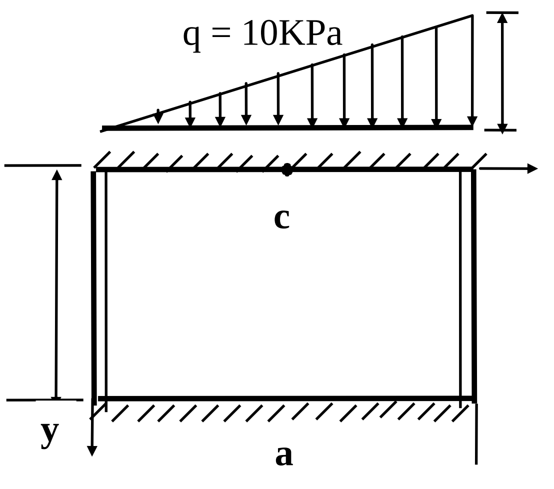

Figure 5. Case 5. Two edges simply supports and two edges fixed subjected to triangular load of q per unit area.

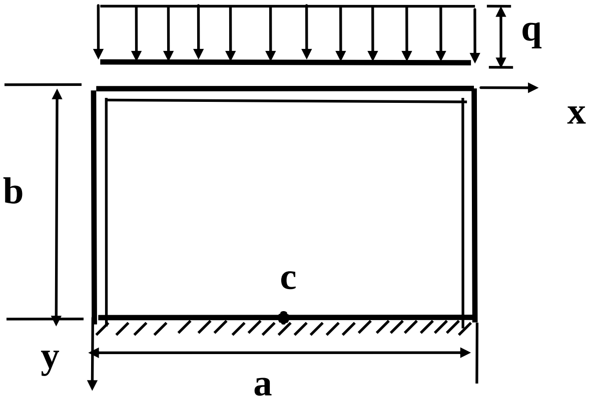

Figure 5. Case 6. Three edges simply supported and the forth edge is fixed under uniform load.

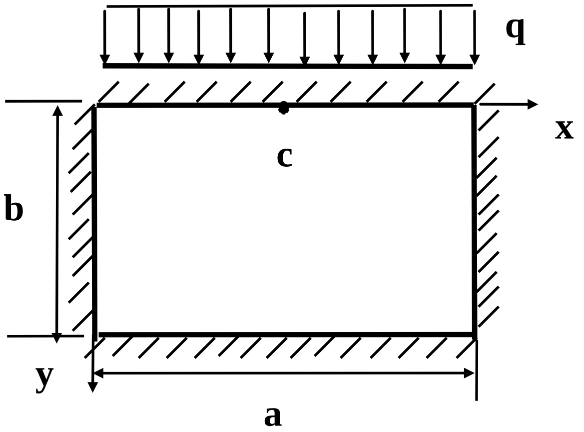

Figure 5. Case 7. Rectangular plate fixed at all sides subjected to a uniform distributed load.

Divide the rectangular plate into 324 elements and by using a computer program the results are shown at a center of plate as shown in table 2.

Table 2. Results of seven cases of orthotropic rectangular plates with different boundary condition under different types of load.

Cases	Nx × Ny Number of grid spacing	Total number of elements	w center m	M_x center N.m	M_y center N.m
Case 1	18 × 18	324	0.89041E-02	0.86104E+04	0.10678E+05
	Analytical solution		0.89106E-02	0.87328E+04	0.10767E+05
Case 2	18 × 18	324	0.44521E-02	0.43052E+04	0.53389E+04
	Analytical solution		0.44553E-02	0.43664E+04	0.53837E+04
Case 3	18 × 18	324	0.13356E-01	0.12916E+05	0.16017E+05
	Analytical solution		0.13366E-01	0.13099E+05	0.16151E+05
Case 4	18 × 18	324	0.32970E-02	0.33849E+04	0.60655E+04
Case 5	18 × 18	324	0.16485E-02	0.16925E+04	0.30328E+04
Case 6	18 × 18	324	0.53148E-02	0.52493E+04	0.77301E+04
Case 7	18 × 18	324	0.27871E-02	0.38018E+04	0.51941E+04

Cases	Nx × Ny Number of grid spacing	Total number of elements	σ_X center Pa	σ_Y center Pa	M_y at point c N.m
Case 1	18 × 18	324	0.35877E+07	0.44490E+07	No point c
	Analytical solution		0.36387E+07	0.44863E+07	No point c
Case 2	18 × 18	324	0.17938E+07	0.22245E+07	No point c
	Analytical solution		0.18193E+07	0.22432E+07	No point c
Case 3	18 × 18	324	0.53816E+07	0.66736E+07	No point c
	Analytical solution		0.54579E+07	0.67296E+07	No point c
Case 4	18 × 18	324	0.14104E+07	0.25273E+07	- 0.12532E05
Case 5	18 × 18	324	0.70520E+06	0.12637E+07	- 0.62662E04
Case 6	18 × 18	324	0.21872E+07	0.32209E+07	- 0.16309E05
Case 7	18 × 18	324	0.15841E+07	0.21642E+07	- 0.11089E05

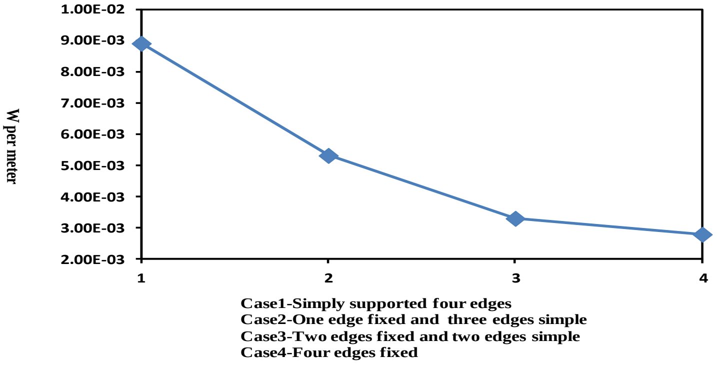

Figure 6. The values of deflection at center of plate for four different cases of boundary condition under uniform distributed load per unit area.

Discussion of effect of different edge conditions of a plate subjected to uniform load:

Deflection effect

From the previous problems. Table 2. Shows the results of deflections for rectangular orthotropic plate simply supported at all sides (Case1), the results of deflections for rectangular orthotropic plate fixed at one edge and the other three edges simply supported (Case6), the results of deflections for rectangular orthotropic plate fixed at two edges and the other two edges simply supported (Case4). Lastly the results of deflections for rectangular orthotropic plate fixed at all edges (Case7).

It is clear that the values of deflections are decreased significantly with increasing the length of fixation as shown in Figure 6.

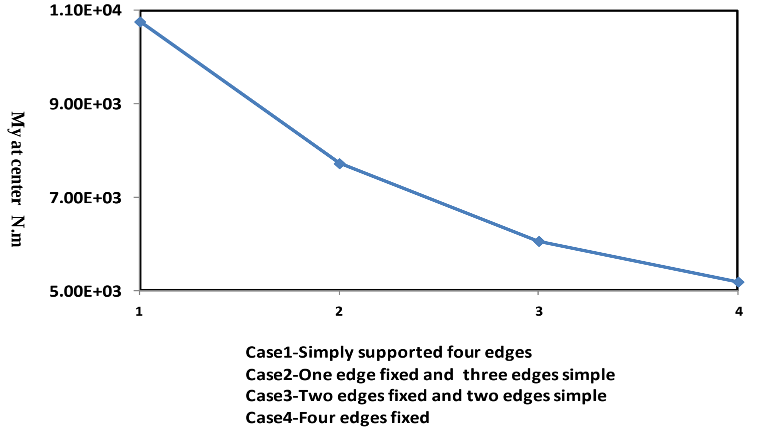

Figure 7. The values of bending moment in y-direction at center of plate for four different cases of boundary condition under uniform distributed load per unit area.

Bending effect

From the previous problems Table 2. Shows the results of bending moments in y-direction for rectangular orthotropic plate simply supported at all sides (Case1), the results of bending moments in y-direction for rectangular orthotropic plate fixed at one edge and the other three edges simply supported (Case6), the results of bending moment in y-direction for rectangular orthotropic plate fixed at two edges and the other two edges simply supported(Case4). Lastly the results of bending moment in y-direction for rectangular orthotropic plate fixed at all edges Case7). It is clear that the values of bending moment in y-direction are decreased significantly with increasing the length of fixation as shown in figure 7. It can be also noted that when the number of elements is increased for rectangular orthotropic plate simply supported at all sides the values of deflection and bending moment increase until approximately reach to the exact solution, while when the number of elements for rectangular orthotropic plate fixed at all sides is increased the values of deflection and bending moment decrease until approximately reach to the exact solution.

Stresses effect:

From the previous problems. Table 2 shows the results of stresses in y-direction for rectangular orthotropic plate simply supported at all sides (Case1), the results of stresses in y-direction for rectangular orthotropic plate fixed at one edge and the other three edges simply supported (Case6), the results of stresses in y-direction for rectangular orthotropic plate fixed at two edges and the other two edges simply supported (Case4). Lastly the results of stresses in y-direction for rectangular orthotropic plate fixed at all edges(Case7).

It is clear that the values of stresses in y-direction are decreased significantly with increasing the length of fixation as shown in Figure 8.

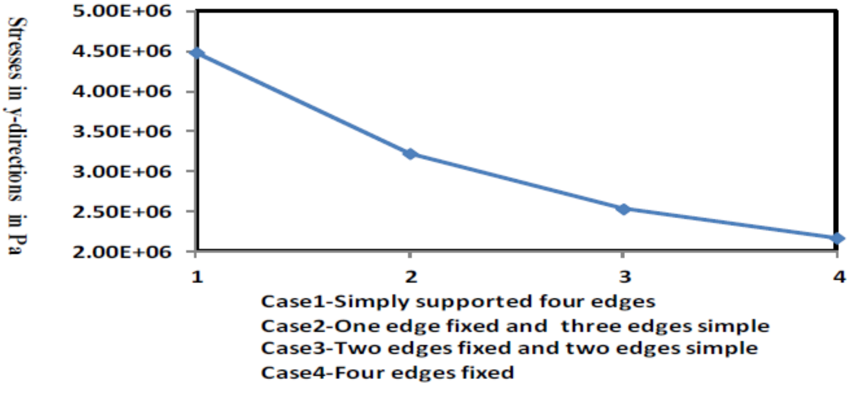

Figure 8. The values of stresses in y-direction at center of plate for four different cases of boundary condition under uniform distributed load per unit area

CONCLUSIONS:

This paper was devoted to the analysis of orthotropic plates with different boundary conditions and for different types of loadings.

Finite difference method was used as a tool for the analysis. Variation of Poisson's ratio and modulus of elasticity in x&y directions were considered.

From this paper which depends on solving of numerical problem of orthotropic plates major findings are listed as follows:

1. Finite difference method is a good tool to solve orthotropic plates
2. If the value of Poisson's ratio increases this will decrease the

value of deflection.

3. The effect of increase in Poisson's ratio has more effect on the bending moments in the long direction of the plate (bending moments in x-direction).

4.The values of deflections are decreased significantly with increasing the length of fixation.

5. The values of bending moment in y-direction are decreased significantly with increasing the length of fixation.

6. When the number of elements is increased for rectangular orthotropic plate simply supported at all sides the values of deflection and bending moment increase until approximately reach to the exact solution, while when the number of elements for rectangular orthotropic plate fixed at all sides is increased the values of deflection and bending moment decrease until approximately reach to the exact solution.

7. The values of stresses in y-direction are decreased significantly with increasing the length of fixation.

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