FEEDBACK STABILISATION FOR IMPERFECTLY KNOWN SINGULARLY PERTURBED SYSTEMS

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ملخص

هذه الورقة تبحث في استقرارية التغذية العكسية الناقصة والفردية كداعم للتعامل (بدرجة تامة) في انظمة التحكم الديناميكية وستظهر في هذه الدراسة بعض المشاكل بسبب العرض والسرعة الديناميكية في عدم الاستقرار والتي سيتم التحكم فيها، حيت تم استخدام نظرية عدم الاستقرار الفردية للتعامل معها.

وباستخدام طريقة رياضية تعرض قياس انظمة فرعية ثنائية يتضمن عدم الاستقرار الاحادي لإنتاج درجة منخفضة عندما يكون عامل التأثير على درجة الصفر. وهذا النظام سيكون وفق الطريقة ليابونوف لقياس درجة لاستقرار والثبات.

ايضا ستبحث هذه الورقة بعضا من المظاهر تقنيات الاستقرار لأنظمة الاضطرابات اللاخطية. ولهذا تم الاستعانة ببعض الأمثلة العملية لإيضاح مخططات الاستقرار. حيث تم انجاز المحاكاة مدعوما بالنتائج المعروضة.

Abstract

This paper is addressed for investigation in feedback stabilisation for imperfectly known, singularly perturbed (full-order) dynamic control systems. Some problems that may arise are the coexistence of slow and fast dynamics in the plant to be controlled. This particular problem can be addressed utilizing singular perturbation theory. The system uncertainties are classified as non-linear perturbations to a known non-linear idealized system. A mathematical model is represented by two time-scale subsystems, which include a scalar singular perturbation parameter that introduce a reduced-order system when the singular perturbation parameter is set to zero. The system analysis is based on Lyapunov's stability technique.

This paper investigates some aspects of stabilisation techniques for nonlinear singularly perturbed systems. Some practical examples are provided to illustrate the stabilization schemes. Simulations have been performed and the results have been presented.

The results clearly indicate that the stability property relating to existence of compact sets, containing the origin which are global uniform attractors for both reduced-order and full-order systems Also, the results show that the effect of the positive scalar parameter increases the singularly perturbed parameter. The results of the practical examples show that when$\varepsilon = 0.01$ tends asymptotically to zero, and when $\varepsilon = 0.5$ becomes unstable

Keywords: Singularly perturbed systems, Control constraints; Global uniform attractor; Output feedback control; singularly perturbed systems; uncertain system

1. Introduction

One of the most basic feedback control problems is that of designing feedback controls to produce trajectories with some desired behaviour for a particular system [1]. As a result of some subsystems that have different time-scale properties, the dynamic behaviour is complex. Some systems may consist of two subsystems for which the dynamics associated with one subsystem is much faster than the dynamics associated with the second subsystem. Such systems can be modelled by incorporating small scalar parameters .These systems are known as singularly perturbed systems [2]. The scalar parameters can occur naturally in systems, as examples for this case, the small time constants. Here, it is supposed that there is only one such parameter, and that some mathematical models can be represented as two subsystems, generally known as a two time-scale or full-order system, which reduces to a system of lower order known as the reduced- order system when singular perturbation parameter is set to zero. The system is generally known as a singularly perturbed system. There are many techniques have been devolved for such systems, formed as set of differential equations that involve a small scalar parameter.

Some systems with actuators and sensors may give rise to the co-existence of slow and fast dynamics in the plant, this need to be controlled.

2. Singular Perturbation Systems

The singular perturbation problems, which depend on the singular perturbation small parameter (ε), may have high dimensionality. Some systems give problems of result: first problems with the analysis, secondly, problems with the design of the feedback controller. The main purpose of two problems are mentioned to solve them for avoiding any problems related to slow and fast dynamics modes. An important modelling task is to separate the system into two subsystems with slow and fast dynamics, which requires insight and ingenuity by the control designer. In the state space, such systems are commonly modelled using the mathematical framework of singular perturbations, with a small parameter (ε), determining the degree of separation between the 'slow' and 'fast' modes of the system. Examples of such systems a rise in many applications include convection-diffusion systems, power systems, scheduling system.

3. Modelling and Analysis of Singularly Perturbed Systems

This section will have a review on singular perturbed systems and related models for this investigation. This section is focussed on the problem of controlling, by feedback, a dynamic system, which depends on small positive scalar parameters, which are known as perturbation parameters. This problem arises from the co-existence of slow and fast dynamics in the plant to be controlled. The analysis for these problems is based on Lyapunov's stability technique:

Here, the system to be investigated is called standard singular perturbation model [3]:

$$\dot{x}(t)=f(t,x(t),z(t),u(t),\varepsilon) \quad (1)$$

εz ̇(t)=g(t,x(t),z(t),u(t),ε) (2)

Where: ε is the singular perturbation parameter.

When the perturbation parameter (ε) is set to zero the system changes dramatically, and generally known as singularly perturbed system .The dynamic qualities of the system are changed, In this case, the subsystem:

ε $\dot{z} = g(t, x, z, \varepsilon)$= degenerates into the algebraic equation:

0=g(t,x,z,ε) (3)

Since there is only one singular perturbation parameter considered here, the principle of the theory developed, in this section involves investigating singular perturbations in two time-scales..

3.1 The model of the Standard Singular Perturbation System

Initially, consider a singular perturbation model that is independent of the control, which has the form:

x ̇(t)=f(t,x(t),z(t))x(t_0)=x^0, x∈R^n (4)

εx ̇(t)=g(t,x(t),z(t))z(t_0)=z^0,x∈R^m (5)

When ε = 0, the dimension of the state equations is reduced from $n + mton$

The reason for this, equation (5) degenerates into the Equation:

$0 = g(t, x, z, \varepsilon)$ (6)

Which an algebraic equation (6). To understand this behaviour system

$f(t, x, z, \varepsilon), x(t_0) = \xi(\varepsilon)$ $=\dot{x}(t)$ (7)

$\varepsilon\dot{x}(t) = g(t, x, z, \varepsilon), z(t_0) = \eta(\varepsilon)$ (8)

Where$\xi(\varepsilon)$ and $\eta(\varepsilon)$ depend onε and $t \in (0, t_0)$is studied. Let $x(t, \varepsilon)$ and $z(t, \varepsilon)$ represent in equation (7) and (8) with variable$z$ of reduced-order model, the system is given by:

$$z = h(t, x,) \quad (9)$$

Thus, substituting (9) into (4), for ε = zero, gives:

$$\dot{x}(t) = f(t, x(t), h(t, x), 0) \quad (10)$$

This is the reduced-order system.

The following example illustrates one singularly perturbed system. In this example, the system is a feedback control system, the gain of feedback control system is large, which involves a high-gain parameter. This high-gain parameter can be expressed as the reciprocal of a small parameter ε, where: $0 < \varepsilon < 1$.

The usage of high-gain parameters, which may become very large, is very common in the feedback control systems. A characteristic approach to the high-gain feedback systems for analysis and design is to model them in the singularly perturbed form. Consider the control system illustrated in Figure.1. In this system, the parameter may be very large.

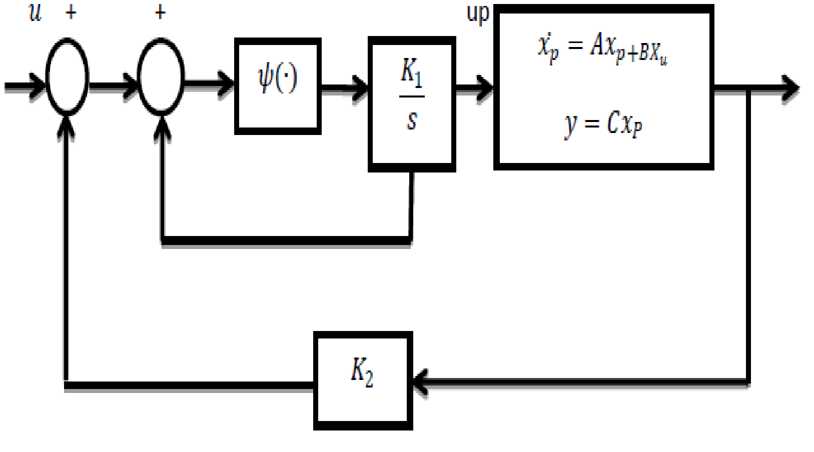

Figure1. Actuator control with high-gain feedback

The state equation for the closed-loop system is:

$$\dot{x}_p = Ax_p + Bu_p \quad (11)$$

$$\frac{1}{K_1}\dot{u}_p = \psi(u - u_p - K_2 C x_p) \quad (12)$$

With $\varepsilon = {}^{1}/_{K_1}$, $x_p = x$, and $u_p = z$,

The model takes the form of equation (6) and (9).

Setting $\varepsilon = 0$, or equationequivalently $K_1 = \infty$,the result will be:

$$\psi(u - u_p - K_2 C x_p) \quad (13)$$

To obtain

$$u_p = u - K_2 C x_p \quad (14)$$

Which is, the unique root since $\psi(.)$ vanishes only at its origin. The resulting reduced model represented by:

x ̇_p=(A-BK_2 C) x_p+B_u) (15)

Equation (15) is the model of the simplified block diagram of Figure.2, where the whole inner loop in the Figure.3is the Simulation diagram

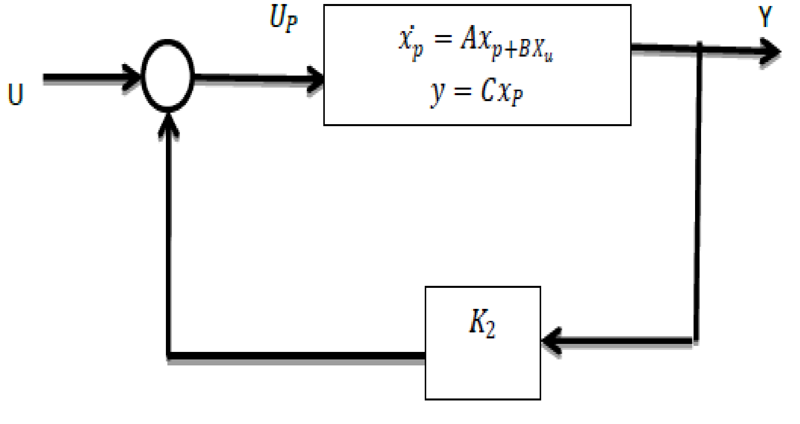

Figure.2. Simplified block diagram of Figure.1

Consider Example 11.2 in Chapter2, Section1 of Khalil [3]

$x_p = [x_1 x_2]^T \epsilon R^2, u_p = z\epsilon R\ , u = 0\ , \varepsilon = \frac{1}{k_1} and k_2 = K, where A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}. B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\ , C = [\ 1 \quad 1\ ]\ and (w) := \frac{w^2}{1+w^2}$.

Simulate the dynamics of the system with k = 4 and initial conditions$(x_1(0) + \ x_2(0) + z(0))$ = (8,-8, 5) for the cases:

(i) ε = 0.01;

(ii) ε = 0.5

4. Simulation Modelling, results and discussion

The simulations performed using MATLAB/Simulink for the proposed dynamic system of the two cases to illustrate the effect of the positive scalar parameter, showing the effect increasing the singularly perturbed parameter.

Case (i), when$\varepsilon = 0.01$:

In the first case, the states tend asymptotically to zero and the system is a good behavior stability to achieve desirable value when used a small parameter values of ε.

Figure.3, shows simulation diagram for performing when ε= 0.01.

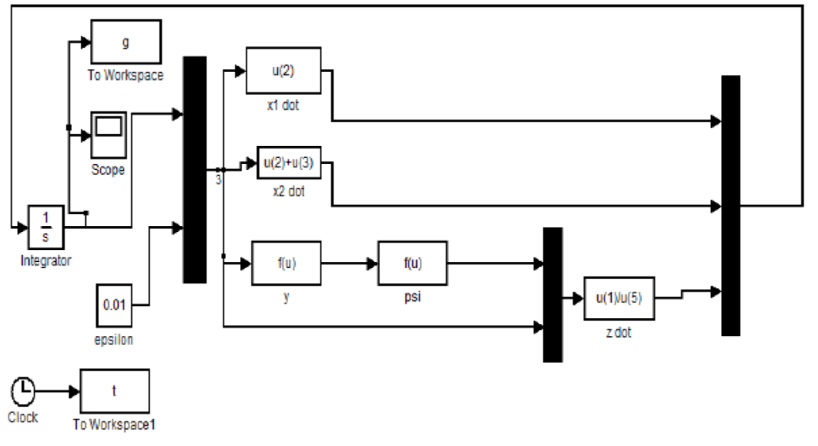

Figure.3. Simulation diagram for performing when ε= 0.01

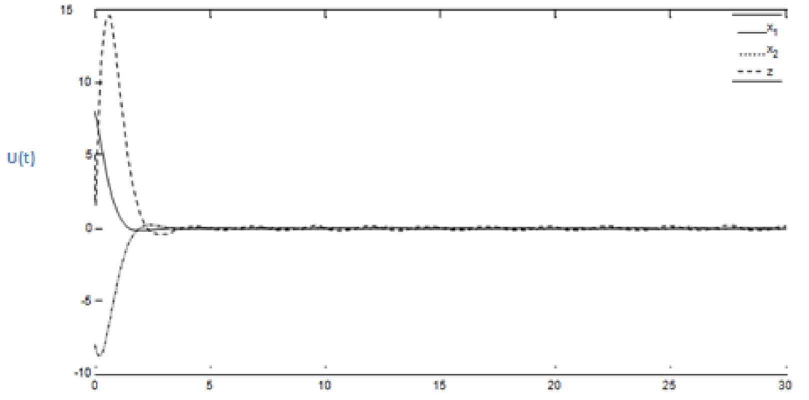

Figure.4.the trajectories of the dynamical system when ε= 0.01

Case (ii), when ε = 0.5.

By applying large parameter value, the states become unstable and the system is a bad behavior stability.

We have noticed that Changing value of parameter effect the stability of system.

Figure.5, shows simulation of the dynamics systems when ε = 0.5. And figure.6. Shows Trajectories of the dynamical system when ε = 0.5, the results displayed below show that system becomes unstable.

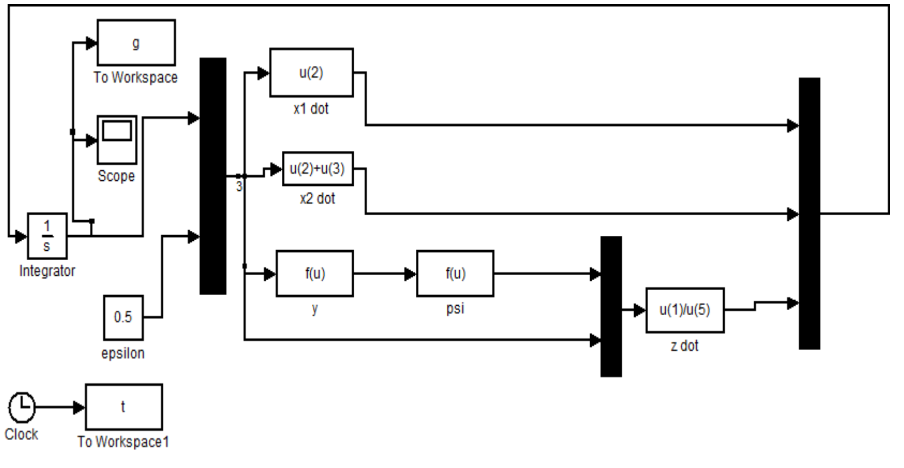

Figure.5. Simulation the dynamics systems when ε = 0.5

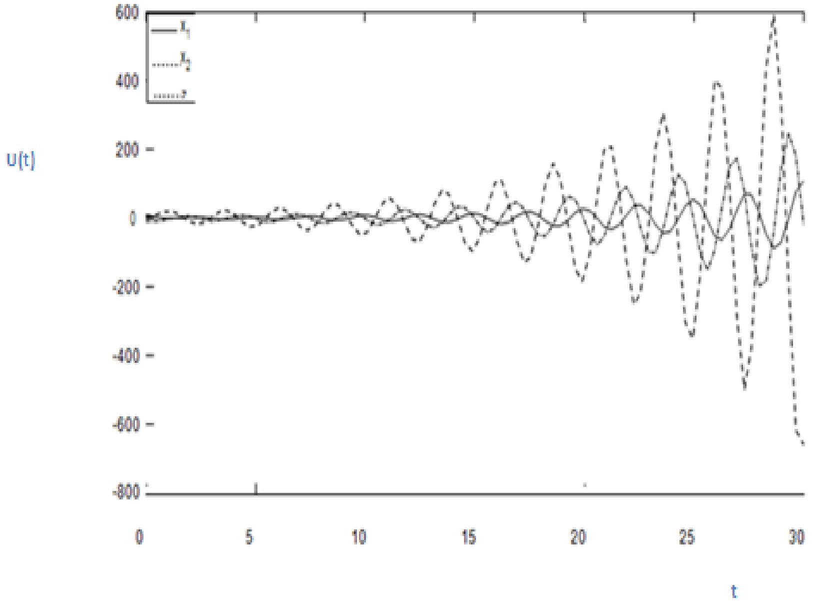

Figure.6. Trajectories of the dynamical system when ε = 0.5

5. Conclusion

In this paper, systems that depend on small scalar parameters were presented. These parameters can give a rise to the coexistence of slow and fast dynamics. A technique is used in this method, which is Lyapunov‟s stability. Modeling Standard Singular Perturbation of System was discussed and some practical examples have been done to investigate the effect of the positive scalar parameter. A more detailed simulation was then carried out using two different value of parameter. Generally these investigations are for illustrating the effect increasing or decreasing the singularly perturbed parameter of system to produce trajectories with some desired behaviour for a particular system.

6. Reference

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